Nonmetric MDS for Sensor Localization

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***Abstract*—Multidimensional Scaling (MDS) has been recently applied to node localization in sensor networks and gained some very impressive performance. MDS treats dissimilarities of pair-wise nodes directly as Euclidean distances and then makes use of the spectral decomposition of a doubly centered matrix of dissimilarities. However dissimilarities mainly estimated by Received Signal Strength (RSS) or by the Time of Arrival (TOA) of communication signal from the sender to the receiver used to suffer errors. From this observation, Nonmetric Multidimensional Scaling (NMDS) based only the rank order of the dissimilarities is proposed in this paper. Different from MDS, NMDS obtain insights into the nature of "perceived" dissimilarities which makes it more suitable to the problem of sensor localization. The experiment on real sensor network measurements of RSS and TOA shows the efficiency and novelty of NMDS for sensor localization problem in term of sensor location-estimated error.**

I. Introduction

Localization of the nodes in wireless sensor networks is one of the main issues in a wireless sensor network. In general, localization algorithms follow the following scheme: anchor unknown distance determination, deriving a nodes position given the anchor distances, and then refinement of the position estimates. Localization algorithms can be divided into two categories: *range-based* and *range-free*. In *range-based* algorithms, nodes estimate their distance to seeds using some specialized hardware. *Range-free* algorithms estimate the location of sensor nodes by, either, exploiting the radio connectivity information among neighboring nodes, or exploiting the sensing capabilities that each sensor node possesses [1]. As a matter of fact, *range-free* techniques are more cost-effective because they do not require sensors to be equipped with any special hardware, but use less information than range-based algorithms. *Range-based* localization depends on the assumption that the absolute distance between a sender and a receiver can be estimated by *Received Signal Strength* (RSS) or by the *Time of Arrival* (TOA) of communication signal from the sender to the receiver. Bahl et al. [2] present RADAR, a radio-frequency (RF) based system for locating and tracking users inside buildings. Bischoff et al. [3] proposed a lightweight localization approach for supporting distance and range queries in ad hoc wireless sensor networks by using RSS to estimate distances. Ward et al. [4] used *Time of Arrival* of signals, while Priyantha et al. [5] and Savvides et al. [6] have used Time Difference of Arrival of signals to estimate distances. And recently Dil et al. [7] proposed a novel range-based algorithm based on the Monte Carlo approach. A review of *range-free* localization schemes from the perspectives of anchor-based and anchor-free solutions are well mentioned in [1]. He et al. [8] proposed a *range-free* algorithm called APIT in which all possible triangles of the seeds are formed. In Gradient algorithm, proposed by Nagpal et al. in [9], the anchor nodes initiate a gradient that self-propagates and allows a sensor node to infer its distance from the anchor. Niculescu and Nath [10] proposed DV-Hop which is similar to Gradient, but uses a different method for estimating the average distance per hop.

II. Related Work

We consider sensor location estimation when sensors measure received signal strength (RSS) or time-of-arrival (TOA) between themselves and neighboring sensors. We use an extensive TOA and RSS measurement campaign in an indoor office from the work in [11]. Some related works are previous ones based on MDS [12][13][14]. MDS-MAP [12] uses a technique from mathematical psychology called Multidimensional Scaling (MDS). The intuition behind multidimensional scaling is simple. Suppose there are N points, suspended in a volume. We don't know the positions of the points, but we do know the distance between each pair of points. Multidimensional scaling is an $O(N^3)$ algorithm that uses the Law of Cosines and linear algebra to reconstruct the relative positions of the points based on the pairwise distances. In this paper, we focus on sensor location estimation using pair-wise RSS or TOA measurements in a wireless network. Consider a network of $N = m + n$ sensors with m reference and n blindfolded sensors ($m \ll n$). We also call those m known-location sensors anchor nodes. The sensor localization problem corresponds to the estimation of sensor coordinates $X = [x_1, x_2, ..., x_N]$, where $x_i \in \Re^2$ (extention of these results to $\Re^3$ is also possible). Suppose that $[x_1, x_2, ..., x_n]$ are unknown-location sensor coordinates and that $[x_{n+1}, x_{n+2}, ..., x_{n+m}]$ are known-location sensor coordinates, we aim to find $[x_1, x_2, ..., x_n]$ based on the estimated distance (or dissimilarity) δ_{ij} between sensor i and j. The estimation of δ_{ij} could be based on RSS or TOA. The next section of the paper describes Classical MDS and Nonmetric MDS in detail. Section IV presents Procrustes Analysis which seeks the isotropic dilation and the rigid translation, reflection and rotation needed to match the estimated coordinate to the true coordinate of sensor locations. The proposed sensor localization algorithm is also presented in Section IV. Section V comes with the experiment on the real measured sensor network. Conclusions are stated in Section

978-1-4244-1653-0/08/$25.00

VI.

III. Multidimensional Scaling - MDS

Suppose a set of N objects is under consideration and between each pair of objects (i, j) there is a measurement δ_{ij} of the "dissimilarity" between the two objects. The typical goal of MDS is to find a low dimensional space, usually Euclidean, in which points in the space represent the objects, one point representing one object, and such that the distances between the points in the space $\{d_{ij}\}$ match as well as possible the original dissimilarities $\{\delta_{ij}\}$ [15].

A. Classical Multidimensional Scaling - CMDS

Classical Multidimensional Scaling treats dissimilarities $\{\delta_{ij}\}$ directly as Euclidean distances and then makes use of the spectral decomposition of a doubly centered matrix of dissimilarities. Formally, a symmetric $N \times N$ matrix $\Delta = [\delta_{ij}]$ is called a dissimilarity matrix if $\delta_{ij} \geq 0$ (nonnegative elements) and $\delta_{ii} = 0$ (zero diagonal elements). From a given dissimilarity matrix Δ, CMDS algorithm constructs a configuration of points in a Euclidean space of specified dimension p [16]. Because the distances in the configuration space are to be Euclidean, we can have

$$\delta_{ij}^2 = d_{ij}^2(X) = \|x_i - x_j\|^2 = (x_i - x_j)^T(x_i - x_j) \quad (1)$$

By writing the squared distances as $d_{ij}^2(X) = x_i^T x_i - 2x_i x_j + x_j^T x_j$ and defining $\Psi = \left[x_1^T x_1, .., x_N^T x_N\right]^T$ [14], the squared distance matrix $D = [d_{ij}^2(X)]_{i,j=1}^N$ now can be written as

$$D = \Psi e^T - 2X^T X + e\Psi^T \quad (2)$$

where $e \in \Re^n$ is a vector of all ones, and $X = [x_1, x_2, ..., x_N]$. Let us define the inner product matrix $B = X^T X$. Matrix B can be rewritten as

$$B = -HDH \quad (3)$$

where $H = I - ee^T/N$ is the centering operator. In order to recover the coordinates from B, CMDS needs to extract the eigenvectors and eigenvalues as follow

$$B = U\Lambda U^T \quad (4)$$

Let $\Lambda_p = diag(\lambda_1, .., \lambda_p)$ be the diagonal matrix of p largest non-zero eigenvalues of B, and $U_p = [u_1, .., u_p]$ is the corresponding eigenvectors. Then the coordinate matrix X is given by $X = U_p \Lambda_p^{1/2}$.

B. Nonmetric Multidimensional Scaling - NMDS

Nonmetric Multidimensional Scaling is a form of MDS that has a slightly less ambitious goal than metric scaling. Instead of attempting to create a configuration of points for which the pairwise distances approximate the original dissimilarities, Nonmetric MDS attempts only to approximate the ranks of the dissimilarities. The standard nonmetric MDS problem can be formalized as follow [17][18][19][20][21][15]. Given a symmetric zero diagonal matrix $\Delta = [\delta_{ij}]$, find $X = [x_1, x_2, ..., x_N] \in \Re^{p \times N}$ such that

$$\forall i, j, k, l \quad \|x_i - x_j\|^2 < \|x_k - x_l\|^2 \Leftrightarrow \delta_{ij} < \delta_{kl} \quad (5)$$

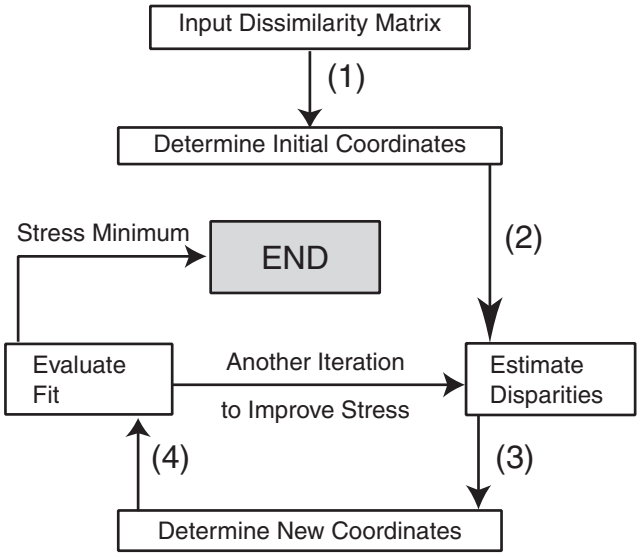

Fig. 1. Shepard-Kruskal algorithm for Nonmetric MDS.

The given dissimilarities δ_{ij} are used to generate a set of derived distances d_{ij}, which are approximately related to the given dissimilarities δ_{ij} by a unknown monotonic increasing function f. We can write $d_{ij} \approx f(\delta_{ij})$ where function f has the property that

$$\delta_{ij} < \delta_{rs} \Leftrightarrow f(\delta_{ij}) < f(\delta_{rs}) \quad (6)$$

The most common approach used to determine the elements d_{ij} and to obtain the coordinates $X = [x_1, x_2, ..., x_N]$ of the objects given only rank order information is an iterative process commonly referred to as the Shepard-Kruskal algorithm (see Fig. 1). The algorithm consists of four steps as follow:

Step 1 - Initial Phase

In this step, we calculate Euclidean distances $d_{ij}^{(0)}$ from an arbitrarily chosen initial configuration $X^{(0)}$ in dimension p, provided that all objects have different coordinates. One might use metric MDS to obtain these initial coordinates.

Step 2 - Nonmetric Phase

The second step or nonmetric phase determines disparities $\tilde{d}_{ij}^{(0)}$ from the distances $d_{ij}^{(0)}$ by constructing a monotone regression relationship between the $d_{ij}^{(0)}$'s and δ_{ij}'s, under the requirement that if $\delta_{ij} < \delta_{kl}$, then $\tilde{d}_{ij}^{(0)} \leq \tilde{d}_{kl}^{(0)}$. This is called the weak monotonic requirement. To obtain the disparities $\tilde{d}_{ij}^{(0)}$, a useful approximation method, *pool-adjacent violators* (PAV) algorithm, is used. The PAV algorithm is described as follows: beginning with the lowest ranked value of δ_{ij}, the adjacent $d_{ij}^{(0)}$ values are compared for each δ_{ij} to determine if they are monotonically related to the δ_{ij}'s. As long as the required monotonic property is true, we assign $\tilde{d}_{ij}^{(0)} = d_{ij}^{(0)}$. Whenever a block of consecutive values of $d_{ij}^{(0)}$ are encountered that violate the required monotonic property the $d_{ij}^{(0)}$ values are averaged together with the most recent non-violator $d_{ij}^{(0)}$value to obtain an estimator $\tilde{d}_{ij}^{(0)}$. Eventually this value $\tilde{d}_{ij}^{(0)}$ is assigned to all

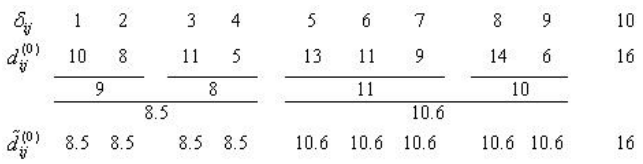

Fig. 2. Illustration of PAV algorithm.

points in the particular block. This procedure is illustrated by the following example [21] in Fig. 2.

Step 3 - Metric Phase

In this step, the spatial configuration of $X^{(0)}$ is altered to obtain $X^{(1)}$. $X^{(1)}$ is selected in such a way that goodness-of-fit is minimized. Common measure used to determine goodness-of-fit is STRESS or SSTRESS, and are given by

$$S = STRESS = \sum_{i<j} \left(\tilde{d}_{ij}^{(0)} - d_{ij}^{(0)}\right)^2 / \sum_{i<j} d_{ij}^{(0)^2} \quad (7)$$

or

$$S = SSTRESS = \left\{ \sum_{i<j} \left(\tilde{d}_{ij}^{(0)^2} - d_{ij}^{(0)^2}\right)^2 / \sum_{i<j} \tilde{d}_{ij}^{(0)^4} \right\}^{1/2} \quad (8)$$

This procedure requires a numerical approximation procedure such as the method of steepest descent. The first step is to place all the coordinates of the points in X in a vector $x = [x_1^T, x_2^T, ..., x_N^T]^T$ vector with Np elements. The stress S is then regarded as a function of x, and is minimized with respect to x in an iterative manner. When the method of steepest descent is used, the update rule is defined as

$$x^{(k+1)} = x^{(k)} - \alpha \frac{\partial S}{\partial x} / \left| \frac{\partial S}{\partial x} \right| \quad (9)$$

where x_k is the vector of coordinates after the k^{th} iteration and α is the step length (refer to [15] for details). From $X^{(1)}$ the new distances $d_{ij}^{(1)}$ can be obtained which are more closely related to the disparities $\tilde{d}_{ij}^{(0)}$ from Step 2.

Step 4 - Evaluation Phase

In the evaluation phase, the goodness-of-fit measure (STRESS or SSTRESS) is used to evaluate whether or not its change as a result of the last iteration is sufficiently small that the procedure is terminated.

IV. Procrustes Analysis and Proposed Algorithm

In this section, we will briefly review procrustes analysis and propose our sensor localization algorithm.

A. Procrustes Analysis

Procrustes analysis seeks the isotropic dilation and the rigid translation, reflection and rotation needed to match one configuration to the other [15]. Suppose a configuration of N points in a q-dimensional Euclidean space, with coordinates given by the $N \times q$ matrix $O_1 = [x_1, .., x_N]^T \in \Re^{N \times q}$, needs to be optimally matched to another configuration of N points in a p-dimensional Euclidean space ($p \geq q$), with coordinates

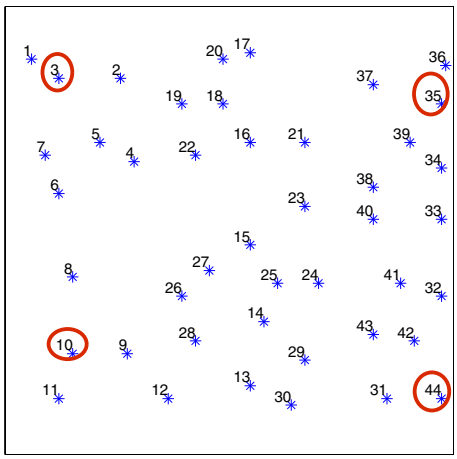

Fig. 3. Experimental sensor locations of 44 nodes with 4 circled anchor nodes (3, 10, 35, 44).

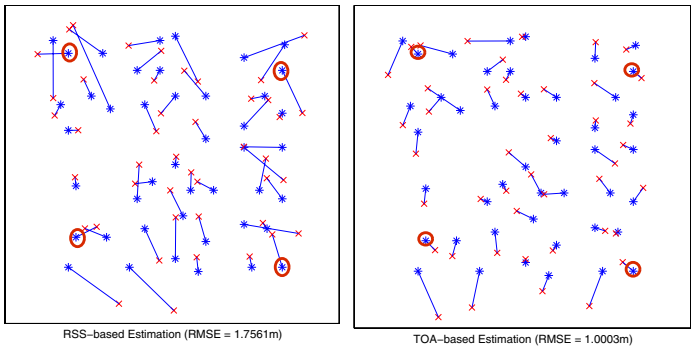

Fig. 4. Sensor Localization based on NMDS is performed using RSS and TOA range measurements from sensor network. True and estimated sensor locations are marked, respectively, by '*' and 'x', while anchor nodes are circled.

given by the $O_2 = [y_1, .., y_N]^T \in \Re^{N \times p}$. In order to make both configurations be at the same p-dimensional space, we firstly add $p - q$ columns of zeros at the right of matrix O_1. It is assumed that the i^{th} point in the first configuration is in a one-to-one correspondence with the i^{th} point in the second configuration. Let the point x_i in the X space be dilated, translated, rotated to new coordinate $\tilde{x}_i = cA^T x_i + b$, the objective function $\Phi(c, A, b)$ needed to be minimized is the new sum of squared distances between points and presented as

$$\Phi(c, T, b) = \sum_{i=1}^{N} (y_i - cA^T x_i - b)^T (y_i - cA^T x_i - b) \quad (10)$$

where the matrix A is orthogonal giving a rigid rotation, vector b is a rigid translation vector, and c is the dilation. The solution to procrustes analysis is well described in [15] and can be summarized as following steps:

- Subtract the mean vectors for the configurations from each of the respective points in order to have the centroids at the origin.
- Find the rotation matrix $A = (O_1^T O_2 O_2^T O_1)^{1/2} (O_2^T O_1)^{-1}$ and rotate the O_1

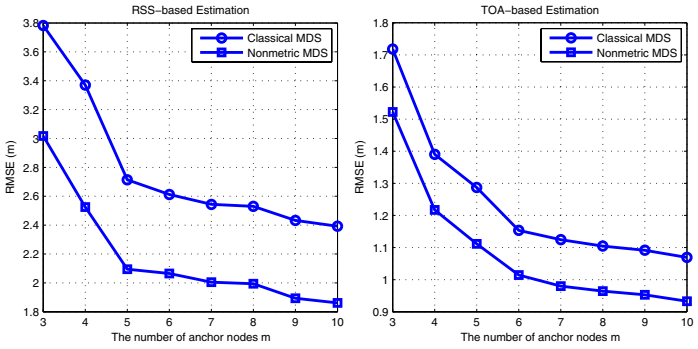

Fig. 5. The performance of MDS and NMDS versus the number of anchor nodes

configuration to O_1A

- Scale the O_1 configuration by multiplying each coordinate by c, where $c = tr(O_1^T O_2 O_2^T O_1)^{1/2}/tr(O_1^T O_1)$

The value of minimized objective function is

$$\Phi_{opt} = tr(O_2^T O_2) - \left\{tr(O_1^T O_2 O_2^T O_1)^{1/2}\right\}^2 /tr(O_1^T O_1) \tag{11}$$

and the translation vector can be calculated as $b = \mu_y - cA^T\mu_x$, where μ_x and μ_y are the centroids of the two configurations.

B. Proposed Sensor Localization Algorithm

Our proposed sensor localization algorithm based on NMDS and procrustes analysis consists of three steps as follow:

Step 1. Construct the dissimilarity (or distance) matrix which contains the dissimilarities between all pairs of sensor nodes based on RSS or TOA.

Step 2. Apply NMDS to the dissimilarity matrix to get the two (or three) dimensional coordinates which is a 2D (or 3D) relative map (in our work, we only experiment on two dimensional map).

Step 3. Given sufficient anchor nodes , we use procrustes analysis to find the rigid rotation matrix A, rigid translation vector b, and the dilation c that match the configuration of absolute anchor nodes to the configuration of relative anchor nodes. And then we use A, b and c to transform the other relative nodes (location-unknown nodes) to form the final estimated map.

V. Experiment on a Measured Network

In our experiment, we use sensor network measurements of received signal strength (RSS) and time-of-arrival (TOA) were made publicly and originally reported in [11]. This data set includes the RSS and TOA range measurements from a network of 44 devices (4 of which are anchor nodes). The measurements were made in an open plan office building, within a 14×14m square area. Four devices near the corners are chosen as reference devices (node 3, 10, 35, 44). The remaining 40 devices are blindfolded devices (see Fig. 3). Before apply MDS or NMDS, we need to construct the dissimilarity (or distance) matrix which contains the dissimilarities between all pairs of sensor nodes based on RSS or TOA.

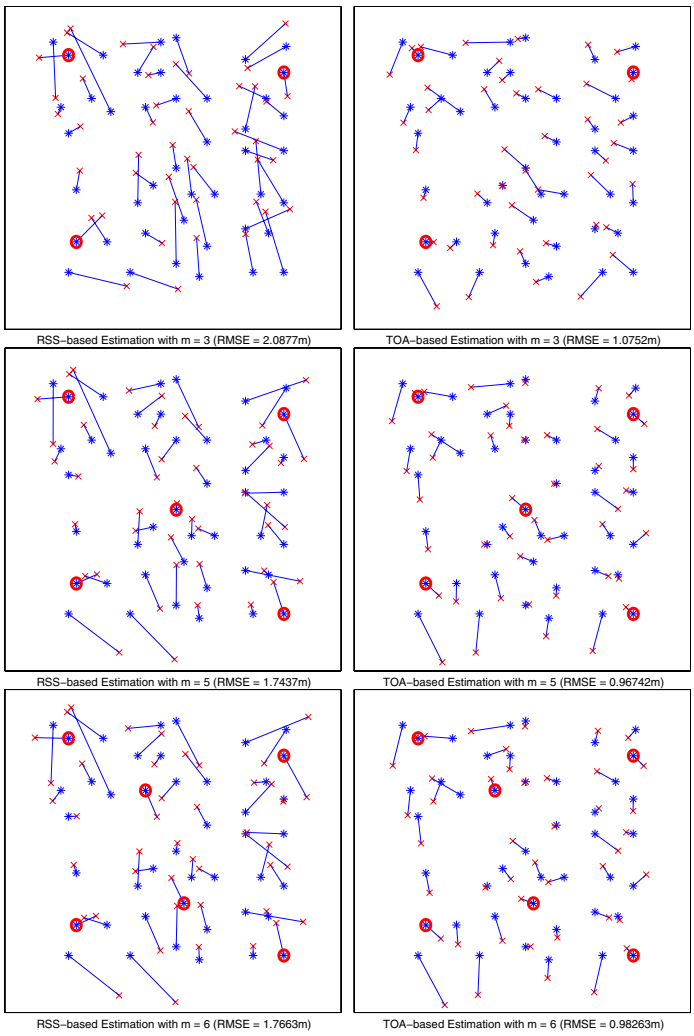

Fig. 6. Sensor Localization based on NMDS is performed using RSS and TOA range measurements from sensor network when the number of anchor nodes $m = 3, 5, 6$ was randomly selected. True and estimated sensor locations are marked, respectively, by '*' and 'x', while anchor nodes are circled.

- *For Distance matrix based on RSS:* We use equation (7) in [11] to estimate range from given received power P_{ij} as:

$$\delta_{ij} = d_0 \left(\frac{P_0}{P_{ij}}\right)^{1/n_p} \tag{12}$$

where P_0(dBm) is the received power in decibel milliwatts at a reference distance d_0 and The path loss exponent n_p is a function of the environment. More details can be referenced in [11].

- *For Distance matrix based on TOA:* We use data matrix of time delay between sensors, in seconds (a 44 by 44 matrix). The (i, j) element is the T_{ij} reported in Section IV of [11], i.e., the average of 10 measured time delays, 5 with the transmitter at i and receiver at j, and 5 with the transmitter at j and receiver at i. The distance between sensor i and j is calculated as

$$\delta_{ij} = |T_{ij}| \times \rho \tag{13}$$

where ρ is speed of propagation (speed of light) in m/s.

In our implementation of NMDS, classical multidimensional scaling is used to choose the initial coordinate of points. For the goodness-of-fit criterion to minimize, we use STRESS criterion (7) which is actually the Stress normalized by the sum of squares of the inter-point distances. The estimated device locations are compared with the actual locations in Fig. 4 with Root Mean Squared Error (RMSE) used to measure location-estimated error. The true and estimated sensor positions are

	MDS	MLE [11]	dwMDS [14]	NMDS
RSS	4.26m	2.18m	2.48m	**1.76m**
TOA	1.85m	1.23m	1.12m	**1.00m**

TABLE I
RMSE OF LOCATION ESTIMATES BASED ON MDS, MLE, DWMDS, AND NMDS IN EXPERIMENTAL NETWORK.

marked by '*' and 'x', respectively, where the lines represent the estimation errors and the anchor nodes are circled with red color. We also compare the performance of the NMDS algorithm to classical MDS, the MLE based solutions from [11], and dwMDS from [14]. Table I summarizes the RMSE of the location estimates based on those methods. It is noted that, we refer the results of MLE and dwMDS in [14]. Next, in order to evaluate the performance of MDS and NMDS versus the number of anchor nodes, we choose randomly $m = 3, .., 10$ sensors as anchor nodes, then apply MDS and NMDS. 50 times of random selection were performed for each value of m and the average RMSE result was recorded. It can be seen from Fig. 5 that NMDS outperforms MDS, and the RMSE of both MDS and NMDS reduces when the number of anchor nodes increases. In Fig. 6, we can see the performance of sensor localization based on NMDS using RSS and TOA range measurements when the number of anchor nodes $m = 3, 5, 6$ was randomly selected. It shows that not only the number of selected anchor nodes affects the performance of NMDS, but also the locations of anchor nodes. For example, the performance of NMDS with $m = 5$ anchor nodes selected as in Fig. 6 (RSS-based RMSE = 1.7437m and TOA-based RMSE = 0.9674m) is better than that of NMDS with $m = 6$ anchor nodes (RSS-based RMSE = 1.7663m and TOA-based RMSE = 0.9826m).

VI. CONCLUSION

We presented a Nonmetric Multidimensional Scaling approach for sensor localization that outperforms MDS with real measurements of received signal strength (RSS) and time-of-arrival (TOA). Because NMDS is based only on the rank order of the dissimilarities, it is more suitable to the problem of sensor localization compared to MDS. The impressive experiment results of NMDS on real measured sensor network exhibits the promising prospects of this approach for the problem of sensor localization. Our future work will be applying this approach for the sensor network with mere connectivity.

ACKNOWLEDGMENT

The authors would like to thank Dr. Neal Patwari for providing the measurements on RSS and TOA of sensor network.

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