Effect of the Modified Channel Matrix on the MMSE V-BLAST System Performance

Trung Q. Duong, Een K. Hong, Sung Y. Lee

School of Electronics and Information – Kyung Hee University
449-701 Suwon, Republic of Korea
dquangtrung79@yahoo.com

***Abstract* - As an important space-time code, V-BLAST (Vertical-Bell Lab Layered Space-Time) code has been studied recently. Zero-Forcing (ZF) and Minimum Mean Square Error (MMSE) are two nulling criteria that are commonly used in the V-BLAST system. In this paper, we propose the new detection algorithm for MMSE criterion that exterminates mutual interferences; hence the BER performance is improved. It can be done by modifying the channel matrix after each detection step. We also perform simulation to validate the analysis.**

Index Terms – **V-BLAST, MIMO, MMSE, wireless communication.**

I. INTRODUCTION

It has been shown in recent research that the deployment of multiple antennas on both sides of a transmitter and receiver provides a larger capacity increase compared to single antennas systems [1], [2], [3]. A multiple-input multiple-output (MIMO) system that employs this trend is the V-BLAST (Vertical Bell Labs Layered Space-Time) architecture proposed in [3]. The structure is designed as a vertically layered coding, where independent code streams (called layers) are assigned with a certain transmit antenna. At the receiver, one way to execute the detection for this system is to use conventional adaptive antenna array (AAA) techniques [3], i.e. linear combination nullling. Nulling is carried out by linearly weighting the received signals in order to meet some relevant performance standard, such as zero-forcing (ZF) or minimum mean square error (MMSE). Zero-forcing was first proposed in [1]. However due to the limitation of pseudo inverse matrix computation when the number of antennas increases, zero-forcing seemed not feasible for real time implementation. To remarkably reduce the computational complexity, a very efficient method utilizing the QR and optimal detection order QR decomposition (called ZF-SQRD) of the channel matrix was proposed in [4]. The problem of noise enhancement through zero-forcing and QR decomposition has been paid attention to. A significant improvement can clearly be seen by including the noise term in the design of the linear weighting vector. This can be done by MMSE detection schemes, where we can trade off between noise and interference proposed in [4].

In this paper, we propose a modification of the channel matrix in MMSE criterion. After detecting the sub-stream, channel matrix will be deflated by zeroing the corresponding column. This is because in MMSE the received vector is not nulled completely. In every detection step, there still remains the interference. In order to minimize as much interference as possible the corresponding channel matrix must be modified, i.e. corresponding column is zeroed, so that the detected sub-stream will not appear in the receive vector. Otherwise, wrongly detected sub-stream will increase the interference for the next step.

The remainder of this paper is organized as follows. In Section II, the V-BLAST system overview is introduced. In Section III, MMSE criterion is briefly reviewed. The effect of the modification of channel matrix is investigated in Section IV. The results are compared in Section V and concluding remark is given in Section VI

II. SYSTEM OVERVIEW

The system is considered with n_T transmit and $n_R \geq n_T$ receive antennas. The data is demultiplexed in n_T data sub-streams of equal length (called layers). These sub-streams are mapped into M-PSK or M-QAM symbols $t_1, t_2, ..., t_{n_T}$ and simultaneously transmitted over n_T antennas. Furthermore, we can use a forward

This research was supported by the MIC(Ministry of Information and Communication), Korea, under the ITRC(Information Technology Research Center) support program supervised by the IITA(Institute of Information Technology Assessment)

error correction code to encode the data sub-streams before mapping. However, it is not addressed in this paper. We will investigate the application under assumption uncoded symbols.

In order to outline the V-BLAST system, one time slot of the time-discrete complex base band model is examined. Let[1] $t = \left[t_1, t_2, ..., t_{n_T} \right]^T$ defines the $n_T \times 1$ vector of transmit symbols, then the corresponding $n_R \times 1$ vector of receive symbols $r = \left[r_1, r_2, ..., r_{n_R} \right]^T$ is given by

$$r = H.t + n \tag{1}$$

In (1), $n = \left[n_1, n_2, ..., n_{n_R} \right]^T$ stands for the white Gaussian noise of variance σ_n^2 observed at the n_R receive antennas while the average transmit power of each antenna is normalized to one i.e. $E\left\{ tt^H \right\} = I_{n_T}$ and $E\left\{ nn^H \right\} = \sigma_n^2 I_{n_R}$

The $n_R \times n_T$ channel matrix H

$$\mathbf{H} = \begin{pmatrix} h_{1,1} & \cdots & h_{1,n_T} \\ \vdots & \ddots & \vdots \\ h_{n_R,1} & \cdots & h_{n_R,n_T} \end{pmatrix} \tag{2}$$

includes $i.i.d$ complex fading gains $h_{j,i}$ expressing the tap gains between transmit antenna i and receive antenna j with unit variance. We presume a flat fading environment, in which the channel matrix H is constant over a frame and changes independently from frame to frame (block fading channel). The distinct gains are assumed to be uncorrelated and are perfectly known in the receiver side

III. MMSE V-BLAST DETECTION

In this section, MMSE criterion in V-BLAST architecture is reviewed. Two major operations are used: nulling and cancellation. With nulling, each sub-stream is regarded as the desired signal and the remaining is considered as interferes.

At each detecting step, all undesired sub-streams are nulled by linearly weighting the receive vector r. The decision statistic y_i of the $i-th$ sub-stream is calculated by multiplying the $i-th$ row of the decorrelating matrix D with the receive vector, given by:

$$y_i = \left(D \right)_i r \tag{3}$$

where $\left(D \right)_i$ represents the $i-th$ row of matrix D corresponding to the criterion in use. In [5] the decorrelating matrix D in the MMSE criterion is chosen to expose the solution for minimizing the problem of this below metric

$$E\left[\left\| t - Dr \right\|^2 \right] = H^H (HH^H + \sigma_n^2 I_{nR})^{-1} \tag{4}$$

The decorrelating matrix is then given by

$$D = H^H (HH^H + \sigma_n^2 I_{nR})^{-1} \tag{5}$$

To attain better performance, nulling always comes together with cancellation. At every detecting step, after the decision the generated version is cancelled from the received signal before moving on to the next stage. The receive vector is linearly weighted with the nulling vector $\left(D \right)_i$ and the result

$$y_i = \left(D \right)_i r = \left(D \right)_i \left(Ht + n \right) = t_i + \widetilde{n_i} \tag{6}$$

y_i is considered as the decision statistic for the $i-th$ sub-stream and $\widetilde{n_i} = \left(D \right)_i n$ is the noise enhancement. By using the quantization operation $Q\left[. \right]$ appropriately, the $i-th$ sub-stream can be estimated likely

$$\hat{t}_i = Q\left[y_i \right] \tag{7}$$

The interference caused by the detected signal $\hat{t}_i$ is now extracted from the receive signal vector r_i

$$r_{i+1} = r_i - \left(H \right)_i \hat{t}_i \tag{8}$$

where $\left(H \right)_i$ is $i-th$ column of the channel matrix.

When symbol cancellation is deployed, the order detection becomes very important to the entire system performance. Let the order set $S = \left\{ k_1, k_2, ..., k_{n_T} \right\}$ be a permutation of the integers $1, 2, ..., n_T$ to specify the detection sequence. Thus the values $y_{k_1}, y_{k_2}, ..., y_{k_{nT}}$ are filtered one by one, the transmit signals $\hat{t}_{k_1}, \hat{t}_{k_2}, ..., \hat{t}_{n_T}$ are estimated and the interference is cancelled out step by step according to equations (5) and (7). In order to obtain the minimum error probability, the optimal order is used. The sub-stream which has the largest post detection signal-to-noise ratio is detected first:

---

[1] *In this paper, (.)^T and (.)^H represents for the matrix transposition and Hermitian transposition, in that order. I_a denotes the $a \times a$ identity matrix*

$$(SNR)_{k_i} = \frac{\left\| (D)_{k_i} (H)_{k_i} \right\|^2 < \left| t_{k_i} \right|^2 >}{\sigma_n^2 \left\| (D)_{k_i} \right\|^2} \qquad (9)$$

where $(D)_{k_i}$ is $k_i - th$ row of decorrelating matrix D, $(H)_{k_i}$ is the $k_i - th$ column of the channel matrix; $|.|$ and $\|.\|$ denote the complex amplitude and the vector norm, respectively. Besides ordering by SNR metric, in order to increase the performance system [5] proposed the SINR metric. The signal to noise and interference of the $k_i - th$ sub-stream to be detected must be maximized.

$$(SINR)_{k_i} = \frac{\left\| (D)_{k_i} (H)_{k_i} \right\|^2 < \left| t_{k_i} \right|^2 >}{\sum_{k_j=1, k_j \neq k_i}^{n_T} \left\| (D)_{k_i} (D)_{k_j} \right\|^2 < \left| t_{k_i} \right|^2 > + \sigma_n^2 \left\| (D)_{k_i} \right\|^2}$$

(10)

IV. EFFECT OF MODIFIED CHANNEL MATRIX ON MMSE CRITERION

When nulling in the MMSE criterion, interference is not nulled absolutely as a trade-off between noise increase and interference decrease is considered. In ZF criterion, at the $k_i - th$ detection stage because of the orthogonal property between $(D)_{k_i}$ and $(H)_{k_i}$ we have the result below

$$(D)_{k_i} r = (D)_{k_i} (Ht + n) =$$

$$= (D)_{k_i} Ht + (D)_{k_i} .n = \alpha.t + (D)_{k_i} n \qquad (12)$$

where $\alpha = [0,0,...,1,...,0]$. The one value appears at $k_i - th$ position and the remaining is zero. On the contrary, in the MMSE the nulling operation is not complete hence the value α is given as

$$\alpha = [\alpha_{k_1}, \alpha_{k_2}, ...\alpha_{k_i}, ..., \alpha_{k_j}, ..., \alpha_{k_{nT}}]$$

where $\alpha_{k_j} \neq 0$. It means that interferences appear at the k_i sub-stream. The main idea here is to reduce as many interference as possible. After detecting k_{i-1} sub-stream the channel matrix is deflated in a way that the $k_{i-1} - th$ column is zero. It is clear that the $k_{i-1} - th$ element of $(D)_{k_i}$ is also zero. As a result, the product α between $(D)_{k_i}$ and $(H)_{k_i}$ is found as

$$\alpha = [0, 0, ...0, \alpha_{k_i}, ..., \alpha_{k_j}, ..., \alpha_{k_{nT}}]$$

In conclusion, the interference from the detected sub-stream is suppressed. These above steps can be summarized as follow

Initialization

$k_i \leftarrow 1$

Recursion

$$D = \left(H^H H + \alpha_n^2 I_{n_T} \right)^{-1} H^H$$

$$y_{k_i} = (D)_{k_i} r$$

$$t_{k_i} = Q(y_{k_i})$$

$$r = r - t_{k_i} (H)_{k_i}$$

$$(H)_{k_i} = 0$$

$$k_{i+1} \leftarrow k_i$$

V. SIMULATION RESULTS

In the simulation, we investigate the bit error rates (BER) for a V-BLAST system deploying uncoded BPSK modulation. **Fig. 1** shows the performance of optimal order ZF and MMSE with and without modified channel matrix. The impact of modified channel matrix can be seen very clearly. MMSE Metric 1 (SINR maximum) gives nearly the same performance compared to ZF case. However when we apply the modified channel matrix into MMSE the BER much more is improved. Metric 1 with modified channel matrix results in a performance improvement of 6dB compared to optimal order ZF. **Fig. 2** shows the performance of the MMSE detection algorithm in all cases. Without modified channel matrix, Metric 1 (SINR maximum) and Metric 2 (SNR maximum) have nearly the same performance. However when applying the modified channel matrix the Metric 1 has the best BER and the difference can be seen obviously.

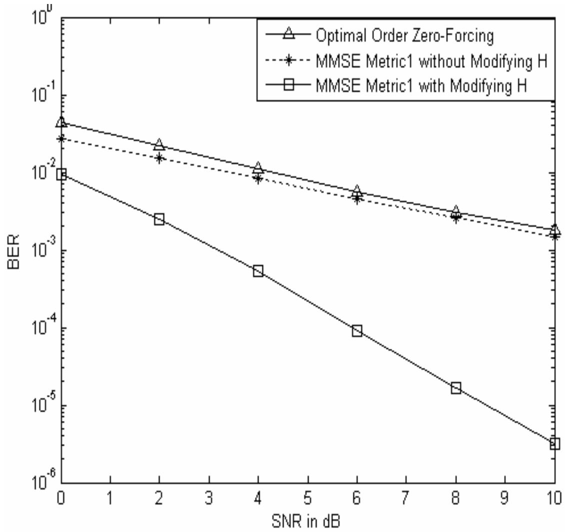

Fig. 1 Simulation V-BLAST with $n_T = 6, n_R = 6$ uncoded BPSK symbols

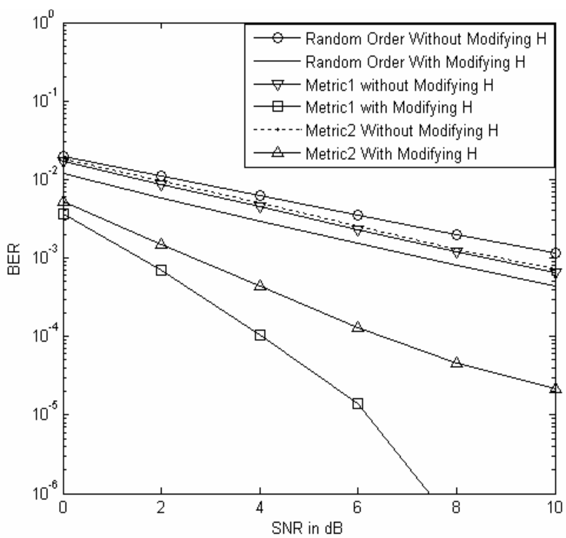

Fig. 2 Simulation MMSE V-BLAST with $n_T = 8, n_R = 8$ uncoded BPSK symbols

VI. CONCLUSION AND DISCUSSION

We have described the influence of modified channel matrix on system performance. Absolutely the BER performance is improved very much because the interference of detected sub-stream is suppressed at the next stage. We also presented simulation results for several scenarios. This proposal can generally be used to in any kind of algorithm.

REFERENCES

[1] G.J.Foschini. "Layered Space-Time Architecture for Wireless Communication in a Fading Environment When Using Multiple Antennas," *Bell Laboratories Technical Journal*, 1(2): 41-59, autumn 1996.

[2] G.J.Foschini. and M.J.Gans. "On the Limits of Wireless Communication in a Fading Environment When Using Multiple Antennas," *Wireless Personal Communications*, 6(3):311-355, 1998.

[3] P. W. Wolniansky. , G.J.Foschini, G. D. Golden, and R. A. Valenzuela. "V-BLAST: An architecture for realizing very high data-rates over the rich-scattering wireless channel,", *Proc. IEEE ISSSE-98*, Pisa, Italia, 30th September 1998.

[4] D.Wubben, J. Rinas, R. Bohnke, V. Kuhn and K. D. Kammeyer. "Efficient Algorithm for Detecting Layered Space-Time Codes," *IEEE Letters*, vol. 37, no. 22, pp. 1348-1350, October 2001.

[5] A. Benjebbour, H. Murata, and S. Yoshida, "Comparison of Ordered Successive Receivers for Space-Time Transmission," *Proc. IEEE Vehicular Technology Conference (VTC)*, USA, Fall 2001.

[6] Ronald Bohnke, Dirk Wubben, Volker Kuhn, Karl-Dirk Kammeyer, "Reduced Complexity MMSE Detection for BLAST Architectures.", *GLOBECOM 2003 – IEEE Global Telecommunications Conference*, vol. 22, no. 1, Dec 2003 pp. 2258-2262

[7] B. Hassibi, "An Efficient Square-Root Algorithm for BLAST", *Proc. IEEE Intl. Conf. Acoustic, Speech, Signal Processing*, Instanbul, Turkey, June 2000, pp. 5-9